Activities with Complex Numbers

Materials
- Large complex plain
- Paper for doing calculations
- Color paper squares (template included)
- Image Grid (template included)
- Human Complex Plane

Activity 1: Basic Arithmetic

Addition
1. Have each person stand on the complex plain and record the complex number that they are standing on.
2. Tell everyone to add 2 to their number. When everyone has done this and double checked their answer. Have everyone walk to their answer.
3. Try doing the same thing by adding i, or 1 + i, or 2 - i, or your favorite complex number.
4. From this we should see that adding a complex number causes a shift in the direction of the number that we are adding.

Multiplication
5. Have everyone go back to where they started. And have everyone multiply their number by 2.
6. When everyone is done and sure that their answer is correct have everyone move together to their answer.
7. Try multiplying by other real numbers. What transformation of the plane is accomplished.
8. Now, go back to the starting position and multiply by i. After everyone moves, determine what transformation was accomplished.
9. Try again with multiplication by -i.
10. Try 1 + i. Try $\frac{1}{2} + \frac{\sqrt{3}}{2}i$. Try multiplication by some other complex numbers.
11. What complex numbers produce a pure rotation of the plane (no dilation)?

Activity 2: Functions of Complex Numbers

1. As a warm-up do the activity on the Spectrum of a Real Function.
   This activity requires the use of the color squares.
2. Now assign everyone a complex number (use numbers of modulus less than 2 in each quadrant).
3. Ask each person to square their number and get a color square of the color corresponding to their answer. They should then put their color square on the answer sheet in the place of their original number. In this way we create the spectrum of the complex function.
4. What do you notice about the spectrum of the function $f(z) = z^2$.
5. Repeat this process with $f(z) = z^3$.
6. Repeat this process with $f(z) = z^2 - 1$. What is the function equal to zero.
7. Repeat this process with $f(z) = z^2 + 1$. What is the function equal to zero.
8. Repeat this process with $f(z) = z^3 + 1$. What is the function equal to zero.
9. Repeat this process with $f(z) = \frac{1}{z}$. What happens at the asymptote.

10. Experiment with other functions.

11. Just for fun, try $f(z) = e^z$. Use a computer. Then try $f(z) = \frac{1}{e^z}$.

The asymptote at the origin is called an essential singularity.

**Activity 3: Julia sets**

**People Version**
1. Have each person stand on a complex number then calculate $f(z) = z^2 - 1$.
2. Have everyone move to their calculated value.
3. Now calculate the same function from where people stand.
4. Move to the new point.
5. Do this several times.
6. After repeating the process several times, have everyone who is still on the board go back to where they started.
7. What do you notice about the process and results.

**Spectrum Version**
1. Calculate the spectrum of $f(z) = z^2 - 1$.
2. Now calculate the spectrum of $f(z) = (z^2 - 1)^2 - 1$.
3. Try $f(z) = (z^2 - 1)^2 - 1$.
4. If you notice a pattern, continue it.
5. The result is the Julia Set for the function $f(z) = z^2 + c$, with $c = -1$. 